BULGARIAN ACADEMY OF SCIENCES
SPACE RESEARCH IN BULGARIA, 3
Sofia • 1980

Interferential Filters in Spectral Instruments without Collimation Optics

K. P. Bakalova

Interferential filters hold an important place among the apparatuses for spectral measurements. It is known by now that their transmission can be described by the function [1]

$$\tau(\lambda, \theta)=\frac{I^{(t)}}{I^{(i)}}=\left(1-\frac{A}{1-R}\right)^2\frac{1}{1+F\sin^2\frac{\delta(\lambda, \theta)}{2}}, \tag{1}$$

where

$$\delta(\lambda, \theta)=\frac{4\pi}{\lambda}nh\cos\theta'+2\varphi \tag{2}$$

is the phase difference between two beams, one of which has had a double inner reflection more than the other. The following symbols have been applied in these expressions:

θ' — angle of refraction in the filter;
n — index of refraction of the intermediate layer of the filter;
h — thickness of the intermediate layer;
φ — phase shift upon inner reflection from metal coatings;
λ — wavelength in vacuum;
$A(\theta)$ — absorption or diffraction of light, accordingly in the metal or poly-layer dielectric reflecting coatings of the filter;
$R(\theta)$ — reflecting capacity of the coatings, and $F=\frac{4R}{(1-R)^2}$.

If we accept the index of refraction of the air for 1, then

$$\cos\theta'=\sqrt{1-\frac{\sin^2\theta}{n^2}},$$

where θ is the angle of fall of the light beam to the filter. Expressions (1) and (2) are valid when a parallel shaft of beams falls on the filter. Consequently, the position of the maximum in the admission band depends on the decline of the beams to the normal to the surface of the filter

$$\lambda_m(\theta)=\frac{2nh}{m-\varphi/\pi}\sqrt{1-\frac{\sin^2\theta}{n^2}},\quad m=1, 2, \ldots,$$

where the decline is expressed by θ.

On increasing the degree of θ the maximum of admission shifts in the direction of the shorter wavelengths. As a matter of principle, the interferential filters are designed for operation with a parallel shaft of light, falling almost horizontally to their surface. For that reason the characteristics con-

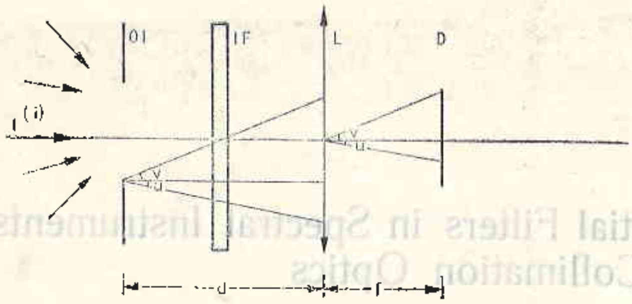

Fig. 1. Optical part of the spectral instrument: OI — optical inlet; IF — interferential filter; L — lens; D — detector

tained in their certificates are for such conditions of operation. In some of the apparatuses for spectral measurements, however, in view of considerations of energy and design, the interferential filters have to operate with non-parallel shafts of light, e. g. in the portable spectrometers for measurements of the spectral reflecting characteristics of some natural formations. The dependence of the position of the maximum in the admission band of the angle of fall limits the angular aperture of the filtered shafts of light [2]. It is not always possible to keep to this limit in practice. In order not to lose the accuracy of measurements, an evaluation of the differences could be made between the performance of interferential filters with a parallel shaft of light, falling normally on their surface, and when working with non-parallel shafts of light at an angle of θ_m. In its most general and simplified form the optical part of the spectral instrument may pe presented as a combination of optical inlet (OI) with a certain aperture through which the measured flow of light enters the instrument; an interferential filter (IF) which admits the light flow in a determined spectral range; a lens (L) which focuses the filtered light on the light-sensitive surface of the detector (D) — a photoelectron converter (Fig. 1). We shall be interested in the signal at the outlet of the detector in both cases — upon lighting the filter with parallel and non-parallel shafts of light.

The light entering the instrument may be considered as a super-position of flat monochromatic waves with length λ, spreading at an angle θ to the optical axis of vision of the system (the normal to the surface of IF, L and D). The intensity of these waves falling on the filter is $I^{(i)}(\lambda, \theta)$.

If it is accepted that the natural objects reflect light diffusely [3], we may consider $I^{(i)}$ as independent on θ. The measurement of $I^{(i)}$ is in a narrow range which is determined by the admission band of the filter. In that range we may consider $I^{(i)}$ as a constant on λ, but its value is determined from its place in the spectrum, i. e. $I^{(i)}(\lambda, \theta) = I_\lambda^{(i)}$. The change in the intensity of each monochromatic wave after its transition through the IF is described by the function (1). Spreading in the afterfilter space are flat monochromatic waves with intensity $L^{(i)}(\lambda, \theta) = I_\lambda^{(i)} \tau(\lambda, \theta)$. We believe that there is no interference between the waves with different λ and that it is possible to neglect the interference between the monochromatic waves spreading at different angles θ in the space after the filter. Then the intensity of the field at each point from the surface of the lens will be the sum of the intensities of all flat monochromatic waves reaching that point. The detector is affected only by these waves for which the angle $\theta \in (0, v)$, where v is determined by the relation $v = \operatorname{arctg} \frac{r_D}{f}$. The energy flow coming to the detector is equal to that of the lens, formed from all waves spreading in the direction θ, with $0<\theta<v$ (if we neglect the losses in the lens). The signal at the output of the detector is proportional to that flow of energy crossing the light-sensitive surface: $dU = S(\lambda, \theta, P)d\Phi$, where $S(\lambda, \theta, P)$ is the sensitivity of the detector, which generally depends on the wave-length λ, the angle of fall θ, and the spot on the light-sensitive surface (point P). In order to simplify, we may assume that S does not depend on the coordinates of the point P and also that it is a constant in the range considered. The dependence on the angle θ is frequently accepted to be a П-like function, i. e.

$$S(\theta) = \begin{cases} S(0), & \theta<\theta_0 \\ 0, & \theta>\theta_0 \end{cases}.$$

Taking into account the above considerations about the output signal of the detector, we come to the expression

$$U = \frac{c}{2} S \int_0^\infty \int_0^{\theta_m} B(\theta) I^{(i)}(\lambda, \theta) \sin\theta \cos\theta \, d\theta \, d\lambda, \tag{3}$$

where: θ_m is the more acute of the angles v and θ_0,

c is the speed of light in vacuum,

$B(\theta)$ is the section of the surface of OI and that of L, when OI is projected at angle θ in the plane of L. Let us assume that the following condition has been fulfilled:

$$r_D = f \frac{r_L - r_{OI}}{d}.$$

In that case $B(\theta)$=constant and the output signal is expressed by

$$U = \frac{c}{2} sB \int_0^\infty I_\lambda^{(i)} \int_0^{\theta_m} \tau(\lambda, \theta) \sin\theta \cos\theta \, d\theta \, d\lambda = \frac{c}{2} sB \int_0^\infty I_\lambda^{(i)} f(\lambda) d\lambda, \tag{4}$$

where θ_m is equal to the smaller of the angles θ_0 and u. The spectral range through which the intensity has been measured is determined from the type of the function $f(\lambda)$. In the above assumptions $f(\lambda)$ is determined from the characteristics of the filter. In other concrete cases it is possible that the quantities S and B will remain under the digit of the integral on θ, i. e. they will also influence the final result from the measurement of $I^{(i)}$. Here is an investigation of the function

$$f(\lambda) = \int_0^{\theta_m} \left(1 - \frac{A}{1-R}\right)^2 \frac{\sin\theta \cos\theta}{1 + F(\theta) \sin^2 \frac{\delta(\lambda, \theta)}{2}} d\theta \tag{5}$$

Table 1

Θ^0	1F, K=1.8		2 F, k=2.6		3 F, k=1.4		4 F, k=1.5		5 F, k=1.15		5 F, k=1.65	
	$\tau_m(\theta)$%	Δ%	$\tau_m(\theta)$%	Δ%	$\tau_m(\theta)$%	Δ%	$\tau_m(\theta)$%	Δ%	$\tau_m(\theta)$%	Δ%	$\tau_m(\theta)$%	Δ%
0	43.0	0.0	25.5	0.0	50.0	0.0	30.5	0.0	25.0	0.0	22.5	0.0
5	42.5	0.0	25.0	0.6	50.0	0.7	30.5	0.9	25.0	0.5	22.5	1.0
10	41.0	0.2	23.0	0.3	49.0	1.0	29.5	0.1	24.5	0.0	21.5	0.4
15	38.5	0.3	20.0	1.3	47.0	0.6	28.5	1.0	24.0	0.5	20.0	2.1
20	34.5	0.9	15.5	0.7	43.5	1.5	26.5	1.6	23.0	0.1	19.0	0.5

for different values of the angle θ_m and for the parameters of the filter. The quantities A and R, and consequently F, are functions of θ, and it is difficult to provide a precise analytical expression. It is clear from (1) that the term $\tau_m(\theta)=\left(1-\frac{A}{1-R}\right)^2$ represents the maximum value of the admission of IF, when lit with a parallel shaft of light falling at an angle of θ to the normal of its surface. This provides an opportunity to determine the dependence $\tau_m(\theta)$ for each filter. Such measurements were carried out for two types of IF by means of Perkin Elmer spectrometer with spectral resolution of 1 nm, which draws the function of admission of the filters from the wavelength. We have the curves of admission for every filter corresponding to lighting up with a parallel shaft of light falling in angles changed every five degrees. The results from the measurements are given in Table 1. The dependences $\tau_m(\theta)$ for all filters approximate well with the function cos $(K\theta)$ at angles $\theta \leq 20°$, where the constant K is different for the different filters and has values of 1.15 to 2.65. The error in the approximation is also given in Table 1.

It may be accepted that the angular aperture of the instruments rarely exceeds 20 degrees, and for that reason it is not necessary to follow the dependence $\tau_m(\theta)$ further. The result is that for almost all filters $\tau_m(35°)<\frac{1}{2}\tau_m(0°)$; a split curve of admission occurring in some bigger values of θ, due to the polarization of light.

It is possible to see the change in the quantity of F with the change of θ in these experimental curves after the formula

$$F(\theta)=\frac{\tau_m(\theta)-\tau(\theta)}{\tau(\theta)\sin^2\frac{\delta(\lambda,\theta)}{2}},$$

which follows directly from (1). The value of $\tau(\theta)$ corresponds to a certain λ. It is assumed that $\varphi=0$, which may occur in filters with reflecting coatings made of polylayer dielectric. It is obvious from (2) that $nh=\frac{\lambda_m(0)}{2}$, if we accept $m=1$. $\lambda_m(0)$ is the length of the wave for which the filter has maximum admission when lit up with a parallel shaft of light. This calculation can be carried out very accurately, because the function $\sin^2\frac{\delta(\lambda,\theta)}{2}$ changes rapidly about its maximum value $\tau_m(\theta)$, but it nevertheless proves that $F(\theta)$ does not change very much. Here are the results of the calculations for one of the filters:

θ^0	0	5	10	15	20
F	122	132	121	134	117

We shall accept that F=const. when calculating the integral (5). The results are given in Table 2. They show that the bigger the index of admission of the intermediate layer of the filter, the less significant the shift of the admis-

Table 2

n	F	k	θ_m^0	λ_m/λ_0	$\Delta\lambda_m/\lambda_0$	$\Delta\lambda_{HW}/\lambda_0$	$f_{\max}\cdot 10^{-2}$
1.5	120	1.5	5	0.9987	0.0013	0.0292	0.3772
1.5	120	1.5	10	0.9950	0.0050	0.0307	1.4258
1.5	120	1.5	15	0.9890	0.0109	0.0366	2.7318
1.5	120	1.5	20	0.9818	0.0182	0.0490	3.7214
2.0	120	1.5	5	0.9990	0.0009	0.0291	0.3776
2.0	120	1.5	10	0.9962	0.0037	0.0300	1.4496
2.0	120	1.5	15	0.9918	0.0082	0.0335	2.9150
2.0	120	1.5	20	0.9860	0.0140	0.0413	4.2363
4.0	120	1.5	5	0.9995	0.0005	0.0291	0.3780
4.0	120	1.5	10	0.9981	0.0019	0.0293	1.4737
4.0	120	1.5	15	0.9959	0.0041	0.0302	3.1360
4.0	120	1.5	20	0.9929	0.0071	0.0324	5.0488
1.5	200	1.5	5	0.9987	0.0013	0.0226	0.3766
1.5	200	1.5	10	0.9950	0.0050	0.0246	1.3923
1.5	200	1.5	15	0.9891	0.0109	0.0317	2.5185
1.5	200	1.5	20	0.9822	0.0177	0.0455	3.2359
1.5	120	2.0	5	0.9987	0.0013	0.0292	0.3760
1.5	120	2.0	10	0.9950	0.0050	0.0307	1.4068
1.5	120	2.0	15	0.9892	0.0108	0.0366	2.6501
1.5	120	2.0	20	0.9826	0.0174	0.0488	3.5310
1.5	120	2.5	5	0.9987	0.0013	0.0292	0.3745
1.5	120	2.5	10	0.9950	0.0050	0.0307	1.3824
1.5	120	2.5	15	0.9895	0.0105	0.0366	2.5472
1.5	120	2.5	20	0.9836	0.0164	0.0486	3.2817

Table 3

θ_m^0	λ, nm / $\Delta\lambda_m$, $\Delta\lambda_{HW}$, nm	400	500	600	700	800
0	$\Delta\lambda_{HW}$	11.66	14.58	17.50	20.41	23.33
5	$\Delta\lambda_m$	0.52	0.65	0.78	0.91	1.04
	$\Delta\lambda_{HW}$	11.68	14.60	17.52	20.44	23.36
10	$\Delta\lambda_m$	2.00	2.50	3.00	3.50	4.00
	$\Delta\lambda_{HW}$	12.28	15.35	18.42	21.49	24.56
15	$\Delta\lambda_m$	4.36	5.45	6.54	7.63	8.72
	$\Delta\lambda_{HW}$	14.64	18.30	21.96	25.62	29.28
20	$\Delta\lambda_m$	7.28	9.10	10.92	12.74	14.56
	$\Delta\lambda_{HW}$	19.60	24.50	29.40	34.30	39.20

sion band, while the half-width of the latter is narrower. The filters with greater reflecting capacity of the coatings R (consequently F is larger also) give narrower admission bands. The influence of the constant K in the dependence of maximum admission of the angle of fall θ: $\tau_m(\theta) = \cos(K\theta)$ over the change

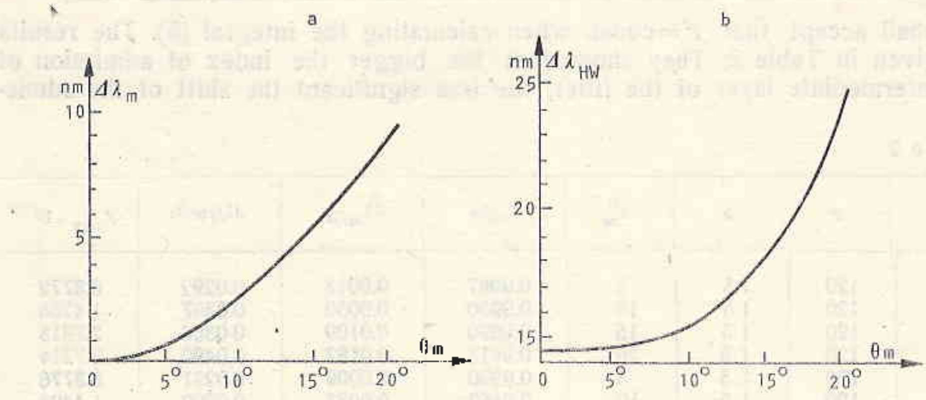

Fig. 2. Dependences of $\Delta\lambda_m(\theta)$ and $\Delta\lambda_{HW}(\theta)$ for $\lambda_0 = 500$ nm

of the admission band is negligible. When choosing the parameters of the filters, the maximum shift occurs for the combination of values: $n = 1.5$; $K = 1.5$; and $F = 120$. The shift of the maximum $\Delta\lambda_m$ and the half-width of admission band for a filter with such parameters on different wavelengths λ and for shafts of light with different angle θ_m are given in Table 3. The shift of the maximum in the admission band $\Delta\lambda_m$ and the half-width of the band increase parallel with the increase of the angle θ_m. The dependences $\Delta\lambda_m(\theta)$ and $\Delta\lambda_{HW}(\theta)$ for $\lambda_0 = 5{,}000$ A are presented on Fig. 2.

It is evident from the above that there exists a certain change in the measured spectral range depending on the working regime of the IF. This change may either be neglected or taken into consideration, depending on the precision necessary for the particular measurements.

References

1. Borne, M., E. Wolf. The Fundamentals of Optics. M., 1973.
2. Seidel, A., G. Ostrowskaya, Y. Ostrovski. Spectrography. M., 1974.
3. Kuchko, A. S. Aerophotography. M., 1974.

Интерференционные фильтры в спектральных приборах без колимирующей оптики

К. П. Бакалова

(Резюме)

Характеристики интерференционных фильтров являются функциями угла падения света. В статье рассмотрена работа интерференционных фильтров со сходящими световыми пучками. Показаны смещение максимума и изменение полуширины полосы пропускания при изменениях углов сходимости пучков в интервале 0—20°.